# Digital PID Controllers
## 2. Implications of the Velocity Form
**Prof. C.R. Nippert**

There are several significant differences between equation 6 and the traditional PID controller shown in equation 1. Let us rewrite the algorithm to simplify the behavior of each action as shown in equation 7

$$\Delta MV = MV_j - MV_{j-1} = K_c\left(\Delta\varepsilon_j + \frac{\Delta t}{\tau_I}\varepsilon_j + \frac{\tau_D}{\Delta t}\left(\varepsilon_j - 2\varepsilon_{j-1} + \varepsilon_{j-1}\right)\right) \qquad (7)$$

Where $\Delta\varepsilon_j = \varepsilon_j - \varepsilon_{j-1}$

We shall focus on the proportional, integral and derivative actions of the PID controller. The terms responsible for each action are shown in the Table below

**Table 1 Proportional, Integral and Derivative Actions of Digital PID Control`**

| Action | Term for the Velocity Form (eqn. 7) | Term from Original PID algorithm (eqn. 1) |
|---|---|---|
| Proportional | $K_c\Delta\varepsilon_j$ | $K_c\varepsilon$ |
| Integral | $K_c\frac{\Delta t}{\iota_I}\varepsilon_j$ | `$\frac{K_c}{\tau_I}\int_{-\infty}^{0}\varepsilon dt$ |
| Derivative | $K_c\frac{\tau_D}{\Delta t}\left(\varepsilon_j - 2\varepsilon_{j-1} + \varepsilon_{j-1}\right)$ | $K_c\tau_D\frac{d\varepsilon}{dt}$ |

There are several important consequences of the velocity form of a PID controller (eqn 7) vs. the original formula (eqn. 1).

1. **"Reset windup" is impossible**
   The integral term in eqn. 1 can accumulate large values if the error persists for a long time. Such accumulated error can adversely influence the controller performance and is known as "reset windup". In the velocity form, the integral term consists of $\varepsilon_j$ or the current value of the controller input (the error). There is no term that can accumulate a large value associated with the integration. "Reset windup", which occurs with analog controllers can cause significant problems with PID controllers and is impossible with digital devices that use the velocity form.

2. **Deviations from theoretical analysis can occur when the set point is changed (derivative kick and proportional kick")**
   There can be a "jump" in the output of a PID controller when the set point is changed. This jump is caused by the behavior of the proportional and derivative actions of equation 7 and is

illustrated in Table 2. Table 2 shows the response of the block diagram from the first section when a change in set point occurs. We assume that the process has a dead time significantly larger than the sampling time (a requirement for good controller operation is that the sampling time MUST be shorter than any process dead time). So the controlled variable does not change during the period shown on the table. The table also assumes the process is under control before the set point is changed, so controlled variable remains at the earlier set point value of 30 throughout.

**Table 2 Response of PID Controller (eqn. 7) to a change in set point**

| Time Period i | Set Point SP | Controlled Variable CV | $\varepsilon_j$ (SP - CV) | Proportional Term $\Delta\varepsilon$ | Integral Term $\varepsilon_j$ | Derivative term $\varepsilon_j - 2\varepsilon_{j-1} + \varepsilon_{j-2}$ |
|---|---|---|---|---|---|---|
| 1 | 30 | 30 | 0 | 0 | 0 | 0 |
| 2 | 30 | 30 | 0 | 0 | 0 | 0 |
| 3 | 30 | 30 | 0 | 0 | 0 | 0 |
| 4 | 40 | 30 | 10 | 10 | 10 | 10 |
| 5 | 40 | 30 | 10 | 0 | 10 | -10 |
| 6 | 40 | 30 | 10 | 0 | 10 | 0 |
| 7 | 40 | 30 | 10 | 0 | 10 | 0 |

The last three columns in Table 2 show the values of the error term in the proportional, integral and derivative terms of the controller. Notice that when the set point is changed, at the 4$^{th}$ time interval, both the value of the proportional term and the derivative terms suddenly change and then return to 0. This spike in both values yields the "kick" of both "proportional kick" and "derivative kick" that changes the controller output (manipulated variable). Notice also, that after the set point change works through the algorithm, both the proportional and derivative terms return to 0. Only the integral term remains nonzero which forces a change in the controller's output. If the integral action is missing, there will be no change in the manipulated variable after the "kick" works its way through the process. The controller will do nothing to bring the controlled variable to the new set point. **For the velocity form to work properly, the integral action must not be turned off ($\tau_I$ must be finite**).

Proportional and derivative kick can be eliminated by modifying equation seven and eliminating the set point from the proportional and derivative terms as shown in equation 8.

$$\Delta MV = MV_j - MV_{j-1} = K_c\left(CV_{j-1} - CV_j + \frac{\Delta t}{\tau_I}\varepsilon_j - \frac{\tau_D}{\Delta t}\left(CV_j - 2CV_{j-1} + CV_{j-2}\right)\right) \qquad (8)$$

If we repeat the change in set point shown in Table 2 with this modified form, we find that the kick is eliminated (see Table 3). BUT the only occurrence of the set point in eqn. 8 is in the integral term so, again, there must always be integral action present when this controller equation is used. This equation exhibits the behavior described by Laplace transforms when the set point is changed.

**Table 3 Response of PID Controller (eqn. 8) to a change in set point**

| Time Period i | Set Point SP | Controlled Variable CV | $\varepsilon_j$ (SP - CV) | Proportional Term ($CV_{j-1}$ - $CV_j$) | Integral Term $\varepsilon_j$ | Derivative term $-\left(\varepsilon_j - 2\varepsilon_{j-1} + CV_{j-2}\right)$ |
|---|---|---|---|---|---|---|
| 1 | 30 | 30 | 0 | 0 | 0 | 0 |
| 2 | 30 | 30 | 0 | 0 | 0 | 0 |
| 3 | 30 | 30 | 0 | 0 | 0 | 0 |
| 4 | 40 | 30 | 10 | 0 | 10 | 0 |
| 5 | 40 | 30 | 10 | 0 | 10 | 0 |
| 6 | 40 | 30 | 10 | 0 | 10 | 0 |
| 7 | 40 | 30 | 10 | 0 | 10 | 0 |